

Correction to the second seminar regarding the Gini coefficient/Lorenz curve

As some of you may have noticed there was a mistake in the suggested solution to Exercise 3 b) from the Supplementary exercises: To the question about the circumstances under which the Gini coefficient is zero, i.e. $G = 0$, I said something of the form: “This corresponds to the case when income is *evenly* distributed in the population, that is, when income follows a uniform distribution.” This statement is of course completely wrong. What I should have written was the following: “This corresponds to the case when income is equally distributed in the population, that is, when everyone has the same income.”

Let X denote the income of a randomly drawn person from the population. If everyone has the same income, denoted $c \in \mathbb{R}_+$, then X follows a particular discrete distribution where $P(X = c) = 1$ and $P(X = x) = 0$ for all $x \neq c$ (this is often referred to as the trivial distribution at c). In such a case the total income, in a population consisting of N individuals, will be Nc . The 100

p % of the population with the lowest incomes consists of (any) pN individuals, all with income c . The total income in this group is therefore pNc , and the value of $L(p)$ becomes the ratio, $pNc/(Nc) = p$. Hence $L(p) = p$ for all p , which implies $G = 0$.

Why do we not get this result (i.e. $G = 0$) with the uniform distribution? As it turns out, if income follows any continuous distribution or any discrete distribution other than the trivial one, there will necessarily be some variation in income in the population. This of course implies a certain degree of income difference, i.e. income inequality, in the population and thus the Lorenz curve and the 45° line will not coincide. Consequently, we must have $G > 0$.